RAMAKRISHNA MISSION VIDYAMANDIRA
(Residential Autonomous College under University of Calcutta)

SECOND YEAR [2014-17]
B.A./B.Sc. THIRD SEMESTER (July – December) 2015
Mid-Semester Examination, September 2015

Date : 14/09/2015 **MATHEMATICS (Honours)**
Time : 11 am – 1 pm Paper : III Full Marks : 50

[Use a separate answer book for each group]

Group – A

Answer **any five** : [5 × 5]

1. a) V be a vector space over a field F and W be a subspace of V. Then show that $\dim V/W = \dim V - \dim W$. [3]

   b) Find a basis for the column space of the matrix $\begin{pmatrix} 0 & 3 & 7 \\ 2 & 1 & 1 \\ 1 & 2 & 4 \end{pmatrix}$. [2]

2. a) If $\{\beta_1, \beta_2, ..., \beta_r\}$ is an orthonormal set of vectors in a Euclidean space V, then for any vector $\alpha \in V$ show that $\|\alpha\|^2 \geq C_1^2 + C_2^2 + ... + C_r^2$, where C_i is the scalar component of α along β_i. [3]

   b) Prove that an orthogonal set of non null vectors in an inner product space V is linearly independent. [2]

3. Let V and W be vector spaces over a field F and V be finite dimensional. If $T: V \to W$ be a linear transformation, show that nullity of $T + \text{rank of } T = \dim V$. [5]

4. Let V and W be finite dimensional vector spaces over a field F and $T: V \to W$ be a linear transformation. Prove that rank of T = rank of matrix of T. [5]

5. A linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^3$ is defined by $T(1,0,0) = (1.3,2), T(1,1,0) = (3,4,0)$ and $T(1,1,1) = (2,1,3)$. Find T and matrix of T relative to the standard basis of $\mathbb{R}^3$. [3+2]

6. V and W be vector spaces over a field F. Prove that a linear transformation $T: V \to W$ is invertible iff T is an isomorphism. [5]

7. Find an orthonormal basis for $\mathbb{C}^3$ using Gram-Schmidt process to the vectors (1,0,i), (2,1,1+i) and (0,i,1). [5]

8. T be a linear operator on $\mathbb{R}^2$ defined by $T(x_1, x_2) = (-x_2, x_1)$. [5]

   a) Find the matrix of T relative to standard basis.

   b) Find the matrix of T in the ordered basis $\{(1,2),(1,-1)\}$.

   c) Prove that for any real c the operator (T – cI) is invertible.

Group – B

9. Answer **any two** questions : [2×5]

   a) A variable plane has intercepts on the co-ordinate axes, the sum of whose squares is a constant K^2. Show that the locus of the foot of the perpendicular from the origin to the plane is $(x^2 + y^2 + z^2)(x^{-2} + y^{-2} + z^{-2}) = K^2$.

   b) A square PQRS of diagonal 2a, is folded along the diagonal PR so that the planes SPR and QPR are at right angles. Show that the shortest distance between SR and PQ is then $\frac{2a}{\sqrt{3}}$.

c) Find the sphere with smallest radius which touches the lines $\frac{x-2}{1}=\frac{y-1}{-2}=\frac{z-6}{1}$ and $\frac{x+3}{7}=\frac{y+3}{-6}=\frac{z+3}{1}$.

d) If a right circular cone of semi-vertical angle θ passes through the x and y axes and also through the line x = y = z, show that $\sec^2\theta = 9-4\sqrt{3}$.

10. Answer **any two** : [2×7·5]

a) A particle moves in one plane under a force which is always perpendicular and towards a fixed straight line on the path. Its magnitude being $\mu \div$(distance from the line)^2. If initially it be at a distance 2a from the line and be projected with a velocity $\sqrt{\frac{\mu}{a}}$ parallel to the line, then prove that the path traced out is a cycloid.

b) Two particles each of mass m, are attached to the ends of an inelastic string which hangs over a smooth pulley; to one of them, A, another particle of mass 2m is attached by means of an elastic string of natural length 'a' and modulus of elasticity 2mg. If the system be supported with the elastic string just unstretched and then released, show that A will descend with acceleration $g\sin^2\sqrt{\frac{g}{2a}}\,t$.

c) A particle is projected vertically upwards with a velocity v_0 in a resisting medium which produces a retardation Kv^2 when the velocity is v. Show that the particle comes to rest at a height $\frac{V^2}{2g}\log\left(1+\frac{v_0^2}{V^2}\right)$ above the point of projection where V is the terminal velocity. Show further that the velocity v_1 of the particle when it again reaches the point of projection is given by $\frac{1}{v_1^2}=\frac{1}{v_0^2}+\frac{1}{V^2}$.

——— × ———